NATURAL CONVECTION HEAT TRANSFER IN AIR ABOUT A HORIZONTAL CIRCULAR NONISOTHERMAL CYLINDER

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Results are presented of a numerical investigation of laminar free convection heat transfer in air about a horizontal circular cylinder in the moderate Grashoff number range ($10^4 \leqslant Gr_m \leqslant 10^9$), for linear and quadratic laws of surface temperature variation.

One meets with heat-transfer processes in natural convection in several areas of contemporary engineering. A topic that has received a good deal of study is steady-state convection near isothermal flat surfaces, and also about isothermal circular cylinders. There is a bibliography on this topic, in particular, in [1, 2].

However, in the actual conditions of operation in specific elements of power equipment, under cooling or heating conditions, considerable temperature differences between the top and bottom points [3] can arise and persist for some. If here the rate of change of temperature is less than 1 deg/sec, then natural convection processes in air can be considered as practically quasistationary [4].

Free laminar convection about a nonisothermal cylinder for one of the possible surface temperature distribution laws was examined in [5]. However, the errors in the numerical method used, and also the simplified mathematical model (boundary-layer approximation, neglect of surface curvature effect), gave rise to inadequate accuracy in the solution obtained, which moreover is discontinuous as φ tends to π. In particular, it is not in satisfactory agreement even with more exact theoretical results for the isothermal case, based on the same flow model [6].

The aim of the present paper is a numerical investigation of natural convection about a horizontal circular cylinder in air (Pr = 0.7), the medium most frequently met with in engineering applications, based on solution of the full Navier–Stokes equations in the Boussinesq approximation, in the range of Grashof number 10^4–10^9, for linear and quadratic surface temperature distribution laws, and various values of temperature differences between the upper and lower points of the axial section.

The mathematical model of the process is the Navier–Stokes equations in the Boussinesq approximation, which, following conversion to dimensionless form and introduction of a new independent variable, reduce to a system of three differential equations with identical structure:

$$\exp(-2\xi)\left\{\frac{\partial}{\partial\varphi}\left(\gamma\Phi\frac{\partial\psi}{\partial\xi}-\frac{\partial\Phi}{\partial\varphi}\right)-\frac{\partial}{\partial\xi}\left(\gamma\Phi\frac{\partial\psi}{\partial\varphi}+\frac{\partial\Phi}{\partial\xi}\right)\right\}=S, \quad (1)$$

where as the desired function Φ we consider, respectively, the axial component of the vorticity, the stream function ψ, and the temperature T. The values of the coefficients γ and S are shown in Table 1. As scales of length, velocity, and temperature we use, respectively, the cylinder radius r_0, the ratio of the diffusivity to the cylinder radius a/r_0, and the difference between the maximum cylinder surface temperature and the air, $t_m - t_f$.

The computational region is the semiannulus of radius $r_0\exp(5) = 148.41\ r_0$. The radius is large enough and the assumption that the independent variables are small on it is well founded.

The problem is solved with the following boundary conditions:

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TABLE 1. Coefficients of Eq. (1)

Φ	γ	S
Ω	$\frac{1}{\mathrm{Pr}}$	$\frac{\mathrm{Gr}_m\mathrm{Pr}}{8}\left(\frac{\partial T}{\partial \xi}\sin\varphi+\frac{\partial T}{\partial \varphi}\cos\varphi\right)\exp(\xi)$
ψ	0	$-\Omega$
T	1	0

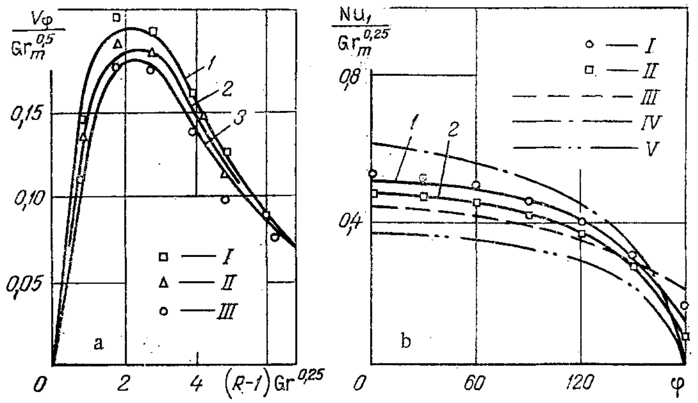

Fig. 1. Comparison of calculated and experimental data for the isothermal case; a) profiles of the tangential component of the velocity vector for $\varphi=90°$ (numerical solution: 1) $\mathrm{Gr_mPr} = 10^5$; 2) 10^6; 3) 10^7; experiment [8]: I) $\mathrm{Gr_mPr} = 4\cdot10^4$; II) $1.9\cdot10^5$; III) $3\cdot10^5$; b) local Nusselt number distribution (present calculation; 1) $\mathrm{Gr_mPr} = 2.76\cdot10^5$; 2) $6.2\cdot10^6$; experiment [8]: I) $\mathrm{Gr_mPr} = 2.76\cdot10^5$; [9]; II) $\mathrm{Gr_mPr} = 6.2\cdot10^6$; III) numerical solution [5]; IV) method of integral realtions [10]; V) numerical solution [3]). The parameter φ is in degrees.

$$\xi=0\ \ \frac{\partial\psi}{\partial\xi}=\psi=0,\ T=T(\varphi);\ \xi=5\ \ T=\Omega=\psi=0;$$

$$\varphi=0,\ \pi;\ \ 0<\xi\leqslant 5\quad \psi=\Omega=\frac{\partial T}{\partial\varphi}=0,$$

where

$$V_r=-\exp(-\xi)\frac{\partial\psi}{\partial\varphi},\ V_\varphi=\exp(-\xi)\frac{\partial\psi}{\partial\xi},$$

$$\Omega=\left[\exp(-\xi)\frac{\partial}{\partial\xi}(\exp(\xi)V_\varphi)-\frac{\partial V_r}{\partial\varphi}\right]\exp(-\xi).$$

The transition from the system of differential equations (1) to a system of finite-difference equations was carried out with the aid of an integrointerpolation method. An explicit scheme was used with a mixed approximation for the convective terms, a combination of directed and central differences. In the solution we used the Seidel method in conjunction with lower relaxation. The boundary conditions for the vorticity at the cylinder surface were determined by the method described in [7].

In order to evaluate the accuracy of the method used we computed the heat transfer in natural convection on an isothermal cylinder. The results obtained agree satisfactorily with the experimental data (Fig. 1). Two possible laws were studied for the variation of dimensionless temperature (or, which is the same thing, dimensionless local temperature head) over the cross section contour: linear and quadratic:

$$T=\frac{\delta-1}{\pi}\varphi+f, \tag{2}$$

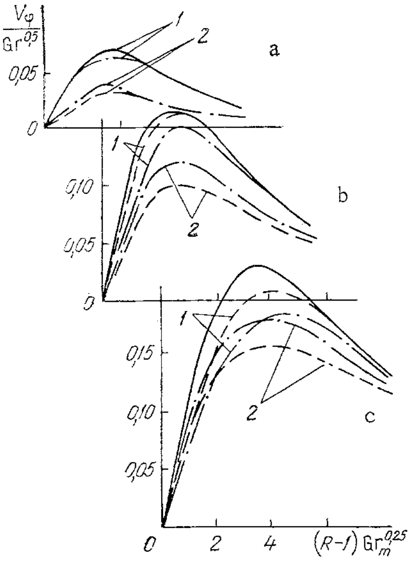

Fig. 2

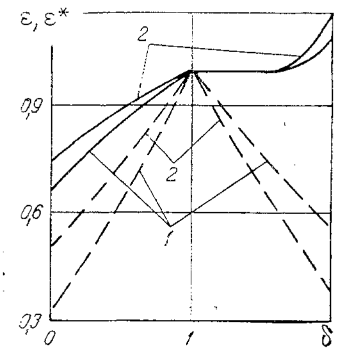

Fig. 3

Fig. 2. Variation of the profile of the tangential component of the velocity vector for various laws of variation and magnitude of the temperature head (($Gr_m Pr = 10^7$); the solid curves are the isothermal law, the broken curves are quadratic, and the dot–dash curves are linear): 1) δ = 0.25; 2) 1.75; a) φ = 30°; b) 90; c) 150.

Fig. 3. The functions ε and ε^* as a function of the law of the variation and the value of the tempeature head (the solid curves are ε; the broken curves are ε^*): 1) linear law; 2) quadratic law.

$$T = \frac{\delta - 1}{\pi^2}\varphi^2 + f, \tag{3}$$

where

$$\delta = (T_u - T_l) + 1, \; f = \begin{cases} 1, & 0 \leqslant \delta \leqslant 1 \\ 2 - \delta, & 1 < \delta \leqslant 2. \end{cases}$$

The value of the constant δ in Eqs. (2) and (3) in the conputations was varied from 0 to 2, which allowed us to span the whole range of temperature differences between the upper (T_u) and lower (T_l) points of the profile, one of which was assumed to be 1.

Analysis of the results of the calculations shows that the nonisothermal nature of the boundary conditions does not change the general character of flow over the cylinder, but it appreciably influences the structure of the free convection flow. Figure 2 shows typical distributions of the tangential component of the velocity vector along the normal to the cylinder surface, for a number of values of the polar angle. It can be seen that, on the whole, the law of variation of the temperature head influences the flow structure less than does the temperature drop. The greatest differences in the distribution of isotherms with respect to the isothermal case were observed for small δ, when the cylinder surface temperature became less than that of the fluid flowing over it: the individual isotherms were closed on an arc of the circle of the cross section lying in the upper semiplane.

From the temperature distribution in the computational region we determined the local and average dimensionless heat fluxes:

$$Q = \frac{qd}{\lambda(t_m - t_f)} = 2\left(\frac{\partial T}{\partial \xi}\right)_{\xi=0},$$

$$\bar{Q} = \frac{qd}{\lambda(t_m - t_f)} = \frac{1}{\pi}\int_0^{\pi} Q d\varphi.$$

TABLE 2. Coefficients of the Polynomials

Parameter	Law of temp. variation over the contour	φ, deg	A_0	$-A_1$	A_2	$-A_3$	A_4
Nu_1	—	—	0,517	$0,203\cdot10^{-2}$	10^{-5}	$0,1245\cdot10^{-7}$	$4,27\cdot10^{-12}$
ε	(2)	—	0,705	2,168	6,231	5,075	1,265
	(3)	—	0,83	2,014	5,64	4,672	1,167
$\frac{Q}{Gr_m^{0,25}}$	(2)	0	0,675	0,963	2,081	1,786	0,455
	(2)	30	0,489	0,839	2,237	1,917	0,488
	(2)	60	0,43	1,216	3,216	2,645	0,673
	(2)	90	0,291	0,98	2,853	2,3	0,57
	(2)	120	0,145	0,704	2,47	2,021	0,5
	(2)	150	0,055	0,733	2,46	1,981	0,498
	(2)	180	—0,0103	0,031	0,657	0,555	0,14
	(3)	0	0,658	1,191	2,704	2,289	0,587
	(3)	30	0,612	1,078	2,557	2,197	0,568
	(3)	60	0,54	1,07	2,59	2,167	0,55
	(3)	90	0,446	1,232	3,105	2,504	0,622
	(3)	120	0,212	0,705	2,36	1,968	0,493
	(3)	150	0,0082	0,46	1,888	1,504	0,365
	(3)	180	—0,015	0,762	1,89	1,3	0,301

With nonisothermal boundary conditions in all the variants of the calculations, the average dimensionless heat flux was reduced in comparison with the isothermal case. Here the factor of decrease in the Grashof number range studied practically depends only on the parameter δ. Therefore, it proved possible to approximate to the computational data, using the least-squares method, for each of the temperature distribution laws considered, in the range of variation of δ most probable for technical applications, by relations of the type

$$\varepsilon=\frac{\overline{Q}_\delta}{\overline{Q}_1}=\sum_{i=0}^{4}A_i\delta^i,\ 0.25\leqslant\delta\leqslant1.75,$$

where $\overline{Q}_\delta$ is the average dimensionless heat flux, corresponding to the fixed value $\overline{Q}_\delta$, and

$$\overline{Q}_1=\sum_{i=0}^{4}A_i\,(\mathrm{Gr}_m\mathrm{Pr})^{0.25(i+1)} \tag{4}$$

is the average dimensionless heat flux for the isothermal case. The coefficients of the polynomials are shown in Table 2, and the relations are shown in Fig. 3. The function ε(δ) depends appreciably on the temperature distribution law for the cylinder surface. Figure 3 also shows the analogous relations for the average Nusselt number

$$\varepsilon^*=\frac{\overline{\mathrm{Nu}}_\delta}{\overline{\mathrm{Nu}}_1}=\varepsilon\pi\Big/\Big(\int_0^\pi T_w d\varphi\Big),\text{where}\overline{\mathrm{Nu}}_1=\overline{Q}_1.$$

It can be seen that with an increase of the temperature head over the contour of the cross section for both laws, over a wide range of variation, the integral mean values of the heat flux density and the temperature head are practically equal.

We note that the calculation of average heat transfer from the parametric relation for the isothermal case, Eq. (4), in which the Grashof number is constructed from the mean-integral temperature head gives acceptable results (the error in the calculations is less than 5%) only in the range $0.9 \leqslant \delta \leqslant 1.25$.

It is of interest to study the influence of the nonisothermal nature of the boundary conditions on the local heat transfer characteristics for the cylinder.

Calculations show that in the range of Grashof number investigated, the group $QGr_m^{-0.25}$ can be approximated, with acceptable accuracy, as a function only of the parameter δ. The results of the calculations were processed by the least-squares method, and for a number of values of the polar angle relations were established of the type

$$Q_\varphi\,\mathrm{Gr}_m^{-0.25}=\sum_{i=0}^{4}A_{\varphi i}\delta^i,$$

where $\varphi=30j;\ j=0,\ 1,\ 2,\ \ldots,\ 6;\ 0{,}25\leqslant\delta\leqslant1.75.$

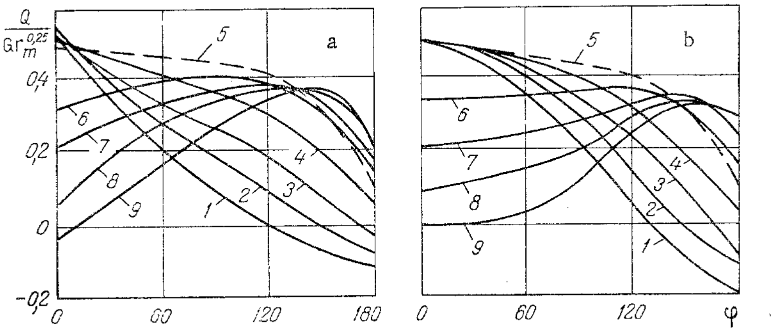

Fig. 4. Local heat transfer on a cylinder with (a) a linear and (b) a quadratic law for variation of the temperature around the cross section: 1) $\delta = 0$; 2) 0.25; 3) 0.5; 4) 0.75; 5) 1; 6) 1.25; 7) 1.5; 8) 1.75; 9) 2.

The coefficients $A_{\varphi i}$ are given in Table 2. With the aid of Eq. (5) we can determine the local heat fluxes at specific points of the contour, and then using some method of interpolation, also at intermediate points. The results of calculations like these are shown in Fig. 4, which also shows, for completeness, the variants $\delta = 0$ and $\delta = 2$, calculated directly from the original equations ($Gr_m = 10^7$). We note that there is a substantial qualitative and quantitative discrepancy from the results obtained for isothermal boundary conditions. For $\delta > 1$ one sees a clearly pronounced maximum in the distribution of $QGr_m^{-0.25}$, whose absolute value decreases with increase of δ, and whose location is displaced toward the top of the half-plane of symmetry. For small δ, beginning with a certain value of φ, the group changes its sign for the reason mentioned above.

Thus, from analysis of the results of the present investigation one can assert that the temperature distribution law around the contour of the cross section of a horizontal circular cylinder has a strong influence on the law of free convection heat transfer. Neglect of this factor, i.e., use in engineering calculations of relations obtained for isothermal boundary conditions, can lead to appreciable errors in determining the local and average heat transfer coefficients, even when there are considerable temperature differences between the upper and lower points.

NOTATION

φ, polar angle, calculated from the lowest point of the contour of cross section of a cylinder; r, radial coordinate; r_0, d, cylinder radius and diameter; $R = r/r_0$; $\xi = \ln(R)$; t_m, maximum temperature on the cylinder surface; t_f, temperature of the fluid at an infinite distance from the cylinder; $T = (t-t_f)/(t_m-t_f)$, dimensionless temperature; Ω, ψ, dimensionless axial component of the vorticity and the stream function; a, ν, λ, β, respectively, the coefficients of diffusivity, kinematic viscosity, heat conduction, and volume expansion of the fluid; g, acceleration due to gravity; $Gr_m = g\beta(t_m-t_f)d^3/\nu^2$, Grashof number; $Pr = \nu/a$, Prandtl number; q, $\bar{q}$, local and average heat fluxes; Q, $\bar{Q}$, local and average dimensionless heat fluxes; α, heat-transfer coefficient; $Nu = \alpha d/\lambda$, Nusselt number. Subscripts: l, local value; m, maximum value; δ, the delta function; 1, isothermal case; w, at the wall; f, far from the cylinder surface.

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FREE-CONVECTION HEAT TRANSFER ON A VERTICAL SURFACE WITH A TEMPERATURE DISCONTINUITY

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Parametric correlations are obtained for calculating heat transfer on a vertical surface with a temperature discontinuity, over a wide range of variation of Prandtl number and for calculating the relations of temperatures at the wall.

The investigation of free convection heat transfer at a wall with various boundary conditions involves problems of singular perturbations of the full Navier—Stokes equations and the energy equation. It has been shown by the method of matched asymptotic expansions that in the first approximation this problem can be considered using the boundary-layer equations [1]. Numerical calculations of free convection on a vertical surface in air [2, 3] agree well with experimental data [4]. The available data in the literature on heat transfer refer to particular cases of temperature discontinuity and to Pr = 0.7.

We consider free convection on a vertical plane surface. On the lower part of the wall ($0 \leqslant x \leqslant x_0$) the temperature is given as T_{W1}, and on the upper part ($x > x_0$) the temperature is T_{W2} ($T_{W1} > T_\infty$, $T_{W2} > T_\infty$). Due to the temperature difference between the wall and the surrounding medium, the motion of the fluid is directed upward, parallel to the wall. Assuming that energy dissipation and the work of compression are negligibly small, we can represent the system of equations of motion and heat transfer in the boundary layer in the form [1]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,\quad u\frac{\partial \vartheta}{\partial x}+v\frac{\partial \vartheta}{\partial y}=a\frac{\partial^2\vartheta}{\partial y^2},$$
$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta\vartheta+\nu\frac{\partial^2 u}{\partial y^2} \tag{1}$$

with the boundary conditions

$$u=0,\ v=0\ \text{for}\ y=0;\ u=0,\ \vartheta=0\ \text{for}\ y\to\infty;$$
$$\vartheta=\vartheta_{c1}\ \text{for}\ x\leqslant x_0;\ \vartheta=\vartheta_{c2}\ \text{for}\ x>x_0. \tag{2}$$

We now transform the system of equations (1), introducing the stream function $\Psi(x, y)$ from the continuity equation:

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